Given a circle, centre $[x_c(t), y_c(t)]$, any point $[x_s, y_s]$on its surface can be described by

$$(x_s - x_c(t))^2 + (y_s - y_c(t))^2 = q^2 \tag{1}$$

where q is the radius of the circle.

We want to find the time to impact on a line of known equation, given the information we had previously about the initial state of the ball, and the point of impact, previously defined as $\mathbf{k} = [k^x, k^y]'$. At the impact point, the tangent slope to the circle is equal to the slope of the line $\mathbf{k}$, which we defined as s, so

$$(x_s - x_c(t)) = -s(y_s - y_c(t)) \tag{2}$$

and we want the ball to hit the line, so from before,

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} k_0^x + k^x \\ k_0^y + sk^x \end{bmatrix} \tag{3}$$

and we know the equations for $x_c(t), y_c(t)$, so

$$\begin{bmatrix} x_c(t) \\ y_c(t) \end{bmatrix} = \begin{bmatrix} v_0^x t + x_0 \\ \frac{1}{2}gt^2 + v_0^y t + y_0 \end{bmatrix}. \tag{4}$$

And you could probably solve this by substitution. In fact, let's do that: Sub 2 into 1, so

$$k_0^y + sk^x - y_c(t) = \frac{q}{\sqrt{s^2+1}} \tag{5}$$

Backsubstitute the right hand side into 2 and expand $x_c(t)$ to solve for k in terms of t:

$$k = \frac{-sq}{\sqrt{s^2+1}} + x_0 + v_0^x t - k_0^x \tag{6}$$

now substitute this into 1 again, expand $y_c(t)$ and solve for t:

$$t^2.\frac{1}{2}g + t.(v_0^y - sv_0^x) + q\sqrt{s^2+1} + y_0 - sx_0 - k_0^y + sk_0^x = 0 \tag{7}$$

... so we get a quadratic, which should give two values of t. Actually, there are up to four solutions, because of time symmetry - we only get two because I left out the $\pm$ in equation 5. So you need to solve the symmetrical quadratic obtained by using the negative solution to 5 to makes sure you get all the solutions. But that should be it.